Que: — (1) Find $\frac{\partial u}{\partial y}$ at (1,1) From the First Principle of

$$u = \frac{1}{\sqrt{x^2+y^2}}$$

Sol^n: — If $u = f(x,y)$, by definition

$$\left(\frac{\partial u}{\partial y}\right)_{(1,1)} = \lim_{K\to 0} \frac{f(1,1+K) - f(1,1)}{K}$$

$$= \lim_{K\to 0} \frac{\frac{1}{\sqrt{1+(1+K)^2}} - \frac{1}{\sqrt{1+1}}}{K} \quad \left\{\because f(x,y) = \frac{1}{\sqrt{x^2+y^2}}\right\}$$

$$= \lim_{K\to 0} \frac{\sqrt{2} - \sqrt{1+(1+K)^2}}{\sqrt{2}K \cdot \sqrt{1+(1+K)^2}}$$

$$= \lim_{K\to 0} \frac{2 - \{1+(1+K)^2\}}{\sqrt{2}K\sqrt{1+(1+K)^2}\left\{\sqrt{2}+\sqrt{1+(1+K)^2}\right\}}$$

$$= \lim_{K\to 0} \frac{-2-K}{\sqrt{2}\sqrt{1+(1+K)^2}\left\{\sqrt{2}+\sqrt{1+(1+K)^2}\right\}}$$

$$= \frac{-2}{\sqrt{2}\cdot\sqrt{2}\cdot 2\sqrt{2}} = -\frac{1}{2\sqrt{2}}$$

Que: - (2) Find f_x, f_y if $f(x,y) = \tan^{-1}\frac{y}{x}$. also Find $f_x(2,1)$

Sol^n: - Here $f(x,y) = \tan^{-1}\frac{y}{x}$

Differentiating w.r to x treating y as Constant

$$f_x = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{-y}{x^2}\right) = \frac{-y}{x^2+y^2}$$

$$\therefore f_x(2,1) = \frac{-1}{2^2+1^2} = \frac{-1}{5}.$$

Differentiating w.r to y treating x as Constant (2)

$$F_y = \frac{1}{1+(y/x)^2}\left(\frac{1}{x}\right) = \frac{x}{x^2+y^2}$$

Que:- (3) If $u = x\sin(y-x)$ prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{u}{x}$$

Sol^n:- we have $u = x\sin(y-x)$ — (1)

Differentiating (1) partially with respect to x (keeping y constant)

we get

$$\frac{\partial u}{\partial x} = \sin(y-x) - x\cos(y-x) \quad \text{— (2)}$$

Differentiating (1) partially with respect to y (keeping x constant)

we get

$$\frac{\partial u}{\partial y} = x\cos(y-x) \quad \text{—— (3)}$$

Adding (2) and (3), we get

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \sin(y-x) = \frac{u}{x}$$